

CS 5000: F23: Theory of Computability

Assignment 4

Vladimir Kulyukin
Department of Computer Science
Utah State University

September 23, 2023

Learning Objectives

1. Regular and Non-Regular Languages
2. Pumping Lemma for Regular Languages
3. Proofs by Contradiction
4. Regular Grammars

Introduction

In this assignment, we'll use the Pumping Lemma for the regular languages and proofs by contradiction to show that some languages are not regular. We'll also investigate regular languages from the perspective of regular grammars. Regular grammars is a conceptual bridge from regular languages and FA to context-free languages and stack machines.

Problem 0 (0 points)

Review the lectures on the regular languages and the Pumping Lemma for the regular languages on Canvas and/or your class notes. I've also included a pdf scan of Ch. 10 from "Formal Language: A Practical Introduction" by A. Brooks Weber on regular grammars. You may want to read it as well.

Problem 1 (1 point)

A language L_1 is a proper sub-language of another language L_2 (formally, $L_1 \subset L_2$) if all strings in L_1 are in L_2 , but there are some strings in L_2 that are not in L_1 . For example, let $L_1 = a^* = \{a^k | k \geq 0\} = \{\epsilon, a, aa, aaa, \dots\}$ and $L_2 = a^*b^* = \{a^ib^j | i \geq 0, j \geq 0\} = \{\epsilon, a, b, aa, bb, ab, aaa, bbb, \dots\}$. Then $L_1 \subset L_2$.

Is there a regular language over an alphabet of 3 symbols (e.g., $\Sigma = \{a, b, c\}$ or $\Sigma = \{0, 1, 2\}$) that has infinitely many non-regular proper sub-languages? If yes, state such a language, some of its proper non-regular sub-languages, and argue that there are infinitely many such sub-languages. If not, argue why such a language cannot exist.

Problem 2 (1 point)

Let Σ be an alphabet and let x be a string over it. Then x^R is the reversal of x . For example, if $\Sigma = \{a, b\}$ and $x = aabb$, then $x^R = bbaa$. For each language below, state if it's regular or not and sketch a proof of your statement.

1. $L = \{a^n b^n c^n | n \geq 0\}$;

2. $L = \{xx^R \mid x \in \Sigma^*\}$, where $\Sigma = \{a\}$;
3. $L = \{xcx^R \mid x \in \Sigma^*\}$, where $\Sigma = \{a, b\}$;
4. $L = \{a^n c^m b^p \mid n + m = p\}$;
5. $L = \{0^n 1^m \mid n \geq m\}$.

Problem 3 (1 point)

Consider a grammar $G = (\{S, A\}, \{a, b\}, S, P)$, where the set of productions P includes the following four productions:

1. $S \rightarrow abS$;
2. $S \rightarrow A$;
3. $A \rightarrow baA$;
4. $A \rightarrow \epsilon$.

Is $L(G)$ regular? Prove your answer.

Problem 4 (1 point)

Let L be the set of all strings consisting of one or more digits, where a digit is a symbol in $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Show that L is regular by constructing a right linear grammar for it.

What to Submit

Save your solutions in `hw04.pdf` and upload it in Canvas.

Happy Thinking!