HW 04

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September 27, 2023

1 Question One

Consider the regular language L over $\Sigma = \{0, 1, 2\}$ such that $L = \{x^\star | x \in \Sigma\}$, then L is a language; its members being all possible combinations of any length of all $x \in \Sigma$.

L is a regular language since there is a FA to describe it:

- $F = \{q_0\}$
- $\Sigma = \{0, 1, 2\}$
- $S = q_0$
- $\delta(q_0, 0) = q_0$, $\delta(q_0, 1) = q_0$, $\delta(q_0, 2) = q_0$
- $Q = q_0$

Let a set of languages G exist such that $G_1 = \{0^i 1^i | i \geq 0\}$, $G_2 = \{(0^i 1^i)(0^i 1^i) | i \geq 0\} \cdots$ $G_n = \{(0^i 1^i)^n | i \geq 0\}$. G_1 is irregular by the proof found in Lecture 5. Then we assume that G_k is irregular. If so, we can show G_{k+1} is irregular because we can only construct a FA to recognize G_{k+1} if and only if we can concatenate a FA recognizing G_k in an epsilon transition with another FA recognizing G_1; which is not existant. By induction, any such $G_i | i \in N$ is irregular.

Each G is also a proper sublanguage since for each $i \in N$ we can construct $(01)^{i+1}$ which is not in G_i but in L, so $x \in G | x = L$. For extra clarity we know every string in G_i is also in L since L is really the Kleene Closure.

Thus there are at least $\aleph_0$ infinitely many such non-regular proper sublanguages of the regular language L.

2 Question Two

2.1 One (adapted from slide notes in Lecture 5)

Consider a minimal DFA M that recognizes L; $L = L(M)$ with k states.

Then consider the string $a^k b^k c^k$; to first recognize a^k we go through $k+1$ states, so we can find a loop in the path taken via δ such that there exists $q = \delta^\star(q, a^i) | i > 0$.

If we pump this loop zero times, then for the string $a^j b^k c^k$, $j < k$; for one or more times, $j > k$; thus $j < k$ or $j > k$ but $j \neq k$, a contradiction from the original definition.

2.2 Two

Any string in this language is an even number of a's, recognized by this FA (thus, is a regular language).

- $F = \{q_0\}$

- $\Sigma = \{a\}$
- $S = q_0$
- $\delta(q_0, a) = q_1, \delta(q_1, a) = q_0$
- $Q = \{q_0, q_1\}$

2.3 Three

Consider a minimal DFA M that recognizes L; $L = L(M)$ with k states. By the pumping lemma each string $x \in L$ such that $|x| \geq k$ is of the form uvw with $|uv| \leq k$, $|v| \geq 1$ and $uv^iw \in L \forall i \geq 0$.

For any such k we create the string $a^kca^k \in L$, and because $|uv| \leq k$ then uv matches at most a^k. So, $u = a^m, v = a^n$ with $m + n \leq k$, and thus $w = a^{k-(m+n)}ca^k$. Additionally, since $|v| \geq 1$, $n \geq 1$.

By pumping v zero times we then have $a^m a^{k-(m+n)}ca^k = a^{k-n}ca^k \notin L$ as $n \geq 1$ so L must be irregular.

2.4 Four

Consider a minimal DFA M that recognizes L; $L = L(M)$ with k states. By the pumping lemma each string $x \in L$ such that $|x| \geq k$ is of the form uvw with $|uv| \leq k$, $|v| \geq 1$ and $uv^iw \in L \forall i \geq 0$.

For any such k we create the string $a^kc^kb^{2k} \in L$ and because $|uv| \leq k$ then uv matches at most a^k. So, $u = a^m, v = a^n$ with $m + n \leq k$, and since $|v| \geq 1$, $n \geq 1$ thus $w = a^{k-(m+n)}c^kb^{2k}$. Additionally, since $|v| \geq 1$, $n \geq 1$.

By pumping v zero times we then obtain $a^m a^{k-(m+n)}c^kb^2k = a^{k-n}c^kb^{2k}$ but then $k-n+k \neq 2k$ as $n \geq 1$, a contradiction; L is irregular.

2.5 Five

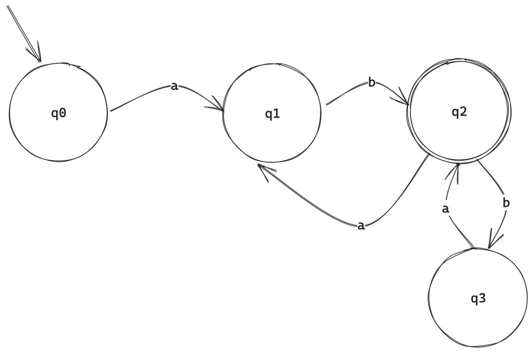
Consider a minimal DFA M that recognizes L; $L = L(M)$ with k states. By the pumping lemma each string $x \in L$ such that $|x| \geq k$ is of the form uvw with $|uv| \leq k$, $|v| \geq 1$ and $uv^iw \in L \forall i \geq 0$.

For any such k we create the string $0^k1^{k-1} \in L$, and because $|uv| \leq k$ then uv matches at most 0^k. So, $u = 0^m, v = 0^n$ with $m + n \leq k$, thus $w = 0^{k-(m+n)}1^{k-1}$. Additionally, since $|v| \geq 1$, $n \geq 1$.

By pumping v zero times we obtain the string $0^m0^{k-(m+n)}1^{k-1} = 0^{k-n}1^{k-1}$ and $k-n$ cannot be greater than $k-1$, a contradiction; L is irregular.

3 Question Three

(pictorial draft) DFA



And:

- $\Sigma = \{a, b\}$
- $Q = \{q_0, q_1, q_2, q_3\}$
- $F = \{q_2\}$
- $S = q_0$
- $\delta(q_0, a) = q_1$, $\delta(q_0, b) = \emptyset$, $\delta(q_1, a) = \emptyset$, $\delta(q_1, b) = q_2$, $\delta(q_2, a) = q_3$, $\delta(q_2, a = q_1)$, $\delta(q_3, a) = q_2$, $\delta(q_3, b) = \emptyset$

We can build a FA that recognizes strings in $L(G)$, so it is regular.

4 Question Four

$G = (\{S\}, \{0, 1, \cdots, 9\}, S, \{S \rightarrow 0S|1S|2S|3S|4S|5S|6S|7S|8S|9S|\epsilon\})$