Assignment Three

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1 Section 2.2

1.1 Question One

1.1.1 b

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\odot$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

1.1.2 c

$\oplus$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$\odot$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

1.2 Question Three

$x = [1], [3], [5], [7]$

1.3 Question Five

$x = [1], [2], [4], [5]$

1.4 Question Eight

$x = [1], [2], [6], [7]$

1.5 Question Eleven

1.5.1 b

$x = [0], [1], [2], [3]$

1.6 Question Fifteen

1.6.1 c

From the Binomial Theorem,

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Then,

$$(a+b)^5 \equiv_5 (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \equiv_5 (a^5 + b^5)$$

since each of the terms $5a^4b, 10a^3b^2, 10a^2b^3, 5ab^4$ are zero as $5 \equiv_5 0$ and $10 \equiv_5 0$.

2 Section 2.3

2.1 Question One

2.1.1 b

$[1], [3], [5], [7]$ since $[7 * 7] = [49] = [1]$, $[5 * 5] = [25] = [1]$, $[3 * 3] = [9] = [1]$, and $[1 * 1] = [1]$.

2.2 Question Two

2.2.1 b

$[2], [4], [6]$ since $[2 * 4] = [8] = [0]$, $[4 * 4] = [16] = [0]$, and $[6 * 4] = [24] = [0]$.

2.3 Question Eight

2.3.1 a

1. $2x = 1$

2. $2x = 3$

3. $2x = 5$

2.3.2 b

Yes, each one is equivalent to 0 when $x = 6$.

2.4 Question Nine

2.4.1 a

By definition, there exists b, the inverse of a, such that $ab = 1$. Assume that a is a zero divisor, then there exists $c \neq 0$ such that $ac = 0$. Then, $(ab)c = 0b \Rightarrow (1)(c) = 0$ implies that $c = 0$, which is a contradiction.

2.4.2 b

By definition, there exists b, with $b \neq 0$ such that $ab = 0$. Assume that a is a unit, then there exists c such that $ac = 1$. Then, $(b)ac = 1b \Rightarrow 0c = b$ implies that $b = 0$, which is a contradiction.

2.5 Question Eleven

By definition of a being a unit, there exists $ay = 1$ with y being an inverse of a. By multiplying our target $(y)ax = (y)b \Rightarrow x = yb$.
To prove this is unique, assume that k and l are solutions of $ax = b$. Then, $ak = b$ and $al = b$. Since a is a unit, by using our previous strategy, $(y)ak = (y)b$ and $(y)al = (y)b$, so $k = yb$ and $l = yb$ and thus $k = l$.

3 Chapter 13

3.1 A2

As $p|c \Rightarrow c = pk$, and $p|c \Rightarrow c = ql$ then $pk = ql$ and thus by Theorem 1.5 since p and q are prime $p|l$ or $p|q$ and $q|p$ or $q|k$, but since p and q are prime, then it must be that only $p|l$ and $q|k$.
Since $p|l$ then $l = mp$ and $c = ql \Rightarrow c = qmp$ and qp is a factor of c.

3.2 A3 (a)

GO = 0715
$715^3 \pmod{2773} = 107$