

Assignment Five

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March 1, 2023

1 Section 4.1

1.1 Question One

1.1.1 d

$$2x^5 + x^4 + 6x^2 + 3x + 2$$

1.2 Question Three

1.2.1 b

```
>>> list(filter(lambda x: x, \
    [f"{a}x^2 + {b}x + {c}" if not a == 0 or not b == 0 else "" \
    for a in range(3) for b in range(3) for c in range(3)]))
```

$$\{ '0x^2 + 1x + 0', '0x^2 + 1x + 1', '0x^2 + 1x + 2', '0x^2 + 2x + 0', '0x^2 + 2x + 1', '0x^2 + 2x + 2', '1x^2 + 0x + 0', '1x^2 + 0x + 1', '1x^2 + 0x + 2', '1x^2 + 1x + 0', '1x^2 + 1x + 1', '1x^2 + 1x + 2', '1x^2 + 2x + 0', '1x^2 + 2x + 1', '1x^2 + 2x + 2', '2x^2 + 0x + 0', '2x^2 + 0x + 1', '2x^2 + 0x + 2', '2x^2 + 1x + 0', '2x^2 + 1x + 1', '2x^2 + 1x + 2', '2x^2 + 2x + 0', '2x^2 + 2x + 1', '2x^2 + 2x + 2' \}$$

1.3 Question Five

Unfortunately, latex'ing long division is not trivial. Sorry in advance.

$$\begin{array}{r}
 3x^2 - 5x + 8 \\
 \hline
 \text{a) } x^2 + x + 1 \overline{) 3x^4 - 2x^3 + 6x^2 - x + 2} \\
 \underline{- 3x^2(x^2 + x + 1)} \\
 -5x^3 + 3x^2 - x + 2 \\
 \underline{+ 5x(x^2 + x + 1)} \\
 8x^2 + 4x + 2 \\
 \underline{- 8(x^2 + x + 1)} \\
 -4x - 6 \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 q(x) = 3x^2 - 5x + 8 \\
 \underline{r(x) = -4x - 6}
 \end{array}$$

$$\begin{array}{r}
 x^3 + 3x^2 + 2x + 3 \\
 \hline
 \text{c) } 2x - 1 \overline{) 2x^4 + x^2 - x + 1} \\
 \underline{- x^3(2x - 1)} \\
 x^3 + x^2 - x + 1 \\
 \underline{- 3x^2(2x - 1)} \\
 11x^2 - x + 1 \\
 \underline{- 2x(2x - 1)} \\
 11x + 1 \\
 \underline{- 3(2x - 1)} \\
 4 \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 q(x) = x^3 + 3x^2 + 2x + 3 \\
 \underline{r(x) = 4}
 \end{array}$$

1.4 Question Six

1.4.1 c

No, since two polynomials of degree $\leq k$ with $k = 2$, under multiplication, could produce a polynomial of degree 4.

1.4.2 d

- Closed under addition (two polynomials with even degrees can only add to polynomials with even degrees)

- Closed under multiplication (two polynomials with even degrees can only multiply to polynomials with even degrees)
- 0_R exists in the set
- For a polynomial a with only even powers, then $a + x = 0_R$ implies that $a = -x$ which will only change coefficients

Seems like a subring (although not rigorously shown) to me!

1.4.3 e

Not a subring, since $x^3 \cdot x = x^4$, with x^3 and x both elements in the described set.

1.5 Question Eleven

```
>>> list(filter(lambda x: ((3 + x) % 9 == 0) and ((3 * x) % 9 == 0), range(9)))
[6]
```

$$(1 + 3x)(1 + 6x) = 1 + 6x + 3x + 18x^2 = 1 + 9x + 18x^2 = 1$$