Building an RC Circuit

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1 The Experiment

The purpose of this experiment was to gain a better understanding of the effects on the voltage over a capacitor as a time-valued function when put in a circuit in series with a resistor. To achieve this goal - and to experimentally verify laws governing the total resistance and capacitance of configurations of resistors and capacitors using a multimeter - we built an "RC circuit" by combining resistors and capacitors from respective smaller-valued components.

Additionally, this lab fulfilled the requirement allowing us to play with dangerously high temperature metal equipment $\ddot\smile$.

1.1 Theory

Here we list some ideas the reader should be familiar with for reference later in the report.

1.1.1 Resistors

Consider n resistors, $r_i \ni i \in [1, n]$ representing the total resistance of the i^{th} resistor, or sub-configuration of resistors, all in parallel. Then, the total resistance, R, of the group is:

$$R^{-1} = \sum_{i=1}^{n} (r_i)^{-1} \tag{1}$$

Consider n resistors, $r_i \ni i \in [0, n]$ representing the total resistance of the i^{th} resistor, or sub-configuration of resistors, all in series. Then, the total resistance, R, of the group is:

$$R = \sum_{i=1}^{n} (r_i) \tag{2}$$

1.1.2 Capacitors

Total capacitance of configurations of capacitors are similar to the inversion of the laws for resistors.

Consider n capacitors, $c_i \ni i \in [1, n]$ representing the total capacitor of the i^{th} capacitor, or sub-configuration of capacitors, all in series. Then, the total capacitance, C, of the group is:

$$C^{-1} = \sum_{i=1}^{n} (c_i)^{-1} \tag{3}$$

Consider n capacitors, $c_i \ni i \in [1, n]$ representing the total capacitor of the i^{th} capacitor, or sub-configuration of capacitors, all in parallel. Then, the total capacitance, C, of the group is:

$$C = \sum_{i=1}^{n} (c_i) \tag{4}$$

1.1.3 The RC Circuit

For a circuit with a resistor of resistance R and capacitor with capacitance C in series, we can model the voltage over the capacitor, V_C, given an initial voltage V_0 and final voltage V_f, as a function of time:

$$V_C(t) = (V_0 - V_f)e^{-\frac{t}{RC}} + V_f \tag{5}$$

1.2 Procedure

The given procedure to exercise our knowledge of equations (1) - (4) if to build both a relatively higher-valued resistor, and capacitor, out of smaller-valued components - by soldering them in series / parallel configurations.

Each pair of students is to produce a resistor and capacitor at a target value (and with a 10% margin for error), determined by seating arrangement. By happenstance, our group was chosen to build:

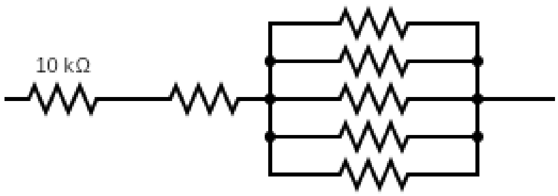
1. A 22 kilo-ohm resistor (22 $k\Omega$)
2. A 1.67 micro-farad capacitor (μF)

out of only 10 $k\Omega$ resistors, and 1 μF capacitors.

I did not record the configuration we used for either. So, assume the following configurations throughout the rest of the lab (pretty sure these were pretty close to our monstrosities):

1.2.1 Building a Resistor

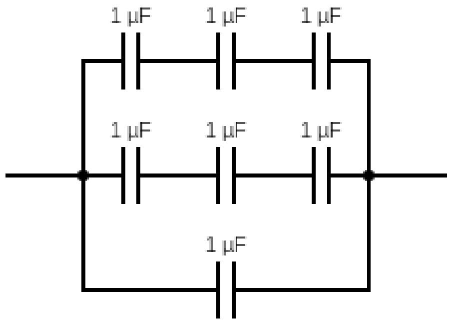
Assume all resistors as $10k\Omega$



In theory, the total resistance measured from the leftmost point to the rightmost is 22 $k\Omega$:

$$\begin{aligned} R &= 10^4 \text{ (leftmost resistor in series (2))} \\ &+ 10^4 \text{ (second leftmost resistor in series (2))} \\ &+ (\frac{5}{10^4})^{-1} \text{ (5 resistors in parallel (1))} \\ &= 2.20 * 10^4 \Omega \end{aligned}$$

1.2.2 Building a Capacitor

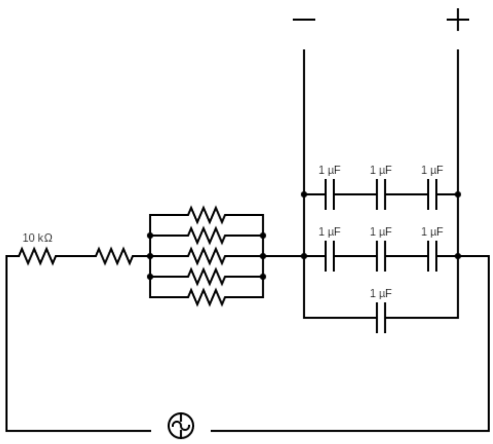


In theory, the total capacitance measured from the leftmost point to the rightmost is 1.67 μF:

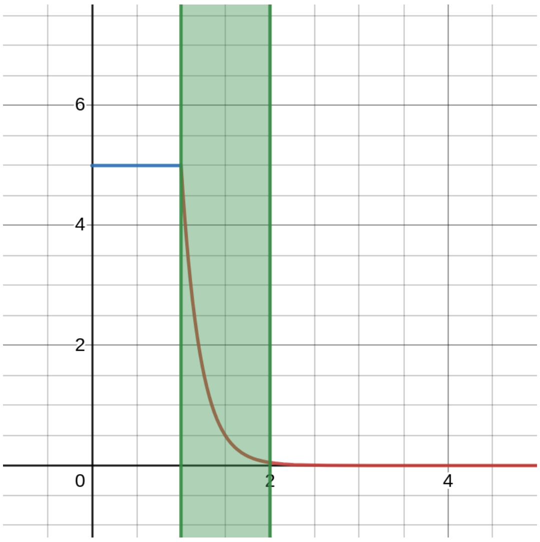
$$
\begin{aligned}
C &= 2(\frac{3}{10-6})^{-1} \text{ (two groups of 3 1-}\mu F\text{ capacitors in series (3) in parallel with)} \\
&+ 10^{-6} \text{ (another 1-}\mu F\text{ capacitor (4))} \\
&\approx 1.67 * 10^{-6} F
\end{aligned}
$$

1.2.3 Determining the RC constant

To measure our RC constant, we connected two voltage probes over V_c (as shown in the diagram below) to a computer-generated positive square wave oscillating at 0.50 Hz with an amplitude of 5V. We then record for 1.5 seconds, polling at 1 kHz, from the time V_C is at 4.95 V (the capacitor has charged) - allowing us to record at least half a second of discharge from the capacitor.



We expect to see that as it discharges, the measured voltage over the capacitor would follow an exponentially decreasing fit, according to the $-\frac{t}{RC}$ term in (5). To find the value of RC we measure the voltage at each discrete time step ($\frac{1}{1000}$ of a second) from near the beginning of the exponential drop to where it reaches stability, and copy those values into a Magic Excel Sheet^{TM}. This region is somewhat shown in the figure below (some values are actually truncated):



The Magic Excel Sheet^{TM} produces a good exponential fit to this data. But, it takes some manual fiddling with the RC value itself to determine the minimum sum of residuals (gradient descent inspired guess and check). The value of RC producing the lowest error by this measure, is our result.

2 Results

2.1 Building a Resistor

The measured resistance (via multimeter) we obtained from our resistor was $21.68k\Omega$.

2.2 Building a Capacitor

The measured capacitance (via multimeter) we obtained from our resistor was $1.78\mu F$.

2.3 The Value Of RC

For our computer determined RC constant, we found it to be $3.72 * 10^{-2}$ s.

3 Discussion

3.1 Building a Resistor

Our target value was $22.00k\Omega$, and we came out with $21.68k\Omega$ - an error of 1.45%.

3.2 Building a Capacitor

Our target value was $1.67\mu F$, and we came out with $1.78\mu F$ - an error of 6.59%.

3.3 The Value of RC

If our resistor and capacitor were exactly on the target value, our RC constant would be $(2.20 * 10^4\Omega)(1.67 * 10^{-6}F) = 3.67 * 10^{-2}$ s.

The RC constant from the measured resistance and capacitance would be $(2.17*10^4\Omega)(1.78*10^{-6}F) = 3.86 * 10^-2$ s.

But, our human-gradient-descent-plus-excel-magic-thanks-computer told us it was $3.72*10^{-2}$ s - a 3.62% error from the theoretical measured value, and 1.36% from the overall "target" value.